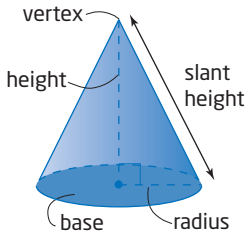
8.4 Surface Area of a Cone

cone

- a three-dimensional object with a circular base and a curved lateral surface that extends from the base to a point called the vertex

A **cone** is a familiar shape to most people. Many of us learn about this shape as children when eating an ice cream cone or snow cone for the first time.

As with a pyramid, the height of a cone is the perpendicular distance from the vertex to the base. The slant height of a cone is the distance from the vertex to a point on the edge of the base.



Investigate

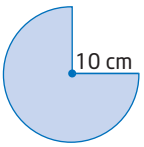
How can you model the lateral area of a cone?

Tools

- construction paper
- scissors
- ruler
- compasses
- tape

Work with a partner.

1. Construct a circle with a radius of 10 cm.

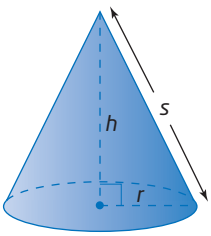


2. Draw two perpendicular radii and cut out and set the smaller sector of the circle aside to use later. What fraction of the circle is the larger piece?

3. Tape the radius edges on the large piece to form a cone. Measure the height, h, of the cone and record it. Measure the radius, r, of the base and record it.

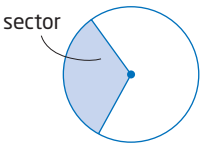
4. Notice that h and r are sides in a right triangle. Calculate the length of the third side, s. How is the length of the third side related to the circle you started with?

5. Calculate the circumference of the base of your cone. What fraction of the circumference of the original paper circle is this?



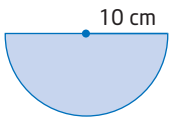
Literacy Connections

A sector of a circle is a part of the circle bounded by two radii and an arc of the circumference.



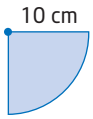
6. The curved surface of the cone is called the *lateral area*. What fraction of the area of your original paper circle is the lateral area of the cone?

7. a) Draw and cut out another circle with radius 10 cm. Draw any diameter and cut along the diameter. Construct a cone using the semicircle for the lateral area.



b) Repeat steps 3 to 6 for this cone.

8. a) Use the smaller sector of the circle you cut out in question 2 to form another cone.



b) Repeat steps 3 to 6 for this cone.

9. Reflect Describe the relationship between the fraction of the circumferences and the fraction of the areas.

You can use proportional reasoning to find the lateral area of a cone. The ratio of the areas is the same as the ratio of the circumferences.

$$\frac{\text{Lateral area of cone}}{\text{Area of circle}} = \frac{\text{Circumference of cone}}{\text{Circumference of circle}}$$

Consider a cone with slant height s and base radius r.

The circumference of the large circle is $2\pi s$ and the circumference of the base of the cone is $2\pi r$.

The area of the large circle is πs^2.

Substitute into the proportion:

$$\frac{\text{Lateral area of cone}}{\text{Area of circle}} = \frac{\text{Circumference of cone}}{\text{Circumference of circle}}$$

$$\frac{\text{Lateral area of cone}}{\pi s^2} = \frac{2\pi r}{2\pi s}$$

$$\frac{\text{Lateral area of cone}}{\pi s^2} = \frac{r}{s}$$

$$\text{Lateral area of cone} = \frac{r}{\cancel{s}_{1}} \times \pi s^{\cancel{2}_{s}}$$

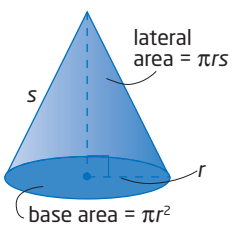
$$\text{Lateral area of cone} = \pi rs$$

The lateral area of a cone with radius r and slant height s is πrs.

The base of a cone is a circle with radius r, so its area is πr^2.

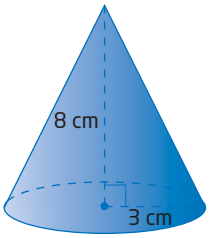
The total surface area of a cone is the sum of the areas of the base and the lateral surface.

$$SA_{\text{cone}} = \pi rs + \pi r^2$$



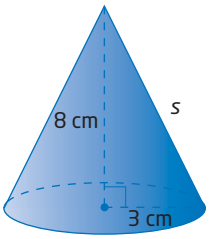
Example Surface Area of a Cone

Calculate the surface area of the cone, to the nearest square centimetre.



Solution

To use the formula for the surface area of a cone, determine the slant height, s.



Use the Pythagorean theorem.

$s^2 = h^2 + r^2$

$s^2 = 8^2 + 3^2$

$s^2 = 64 + 9$

$s^2 = 73$

$s = \sqrt{73}$

$s \doteq 8.5$

The slant height of the cone is about 8.5 cm.

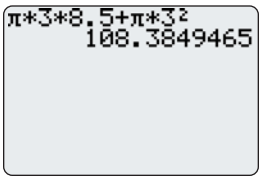
Now, use the formula for the surface area of a cone.

$SA_{\text{cone}} = \pi rs + \pi r^2$

$= \pi(3)(8.5) + \pi(3)^2$

$\doteq 108$

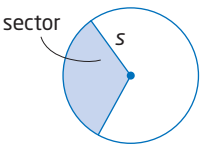
C π × 3 × 8.5 + π × 3 x^2 =



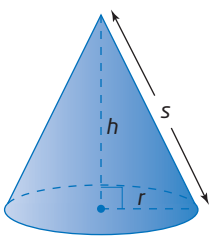
The surface area of the cone is approximately 108 cm^2.

Key Concepts

- The surface area of a cone consists of the lateral area and the area of the circular base.
- The lateral area is formed by folding a sector of a circle. The radius of the circle used becomes the slant height, s, of the cone formed. The area of this curved surface is πrs, where r is the radius of the base of the cone.

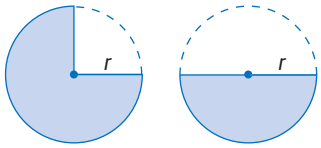


- The area of the circular base is πr^2.
- The formula for the surface area of a cone is $SA_{\text{cone}} = \pi rs + \pi r^2$.
- When you know the radius, r, and height, h, of a cone, you can determine the slant height, s, using the Pythagorean theorem.

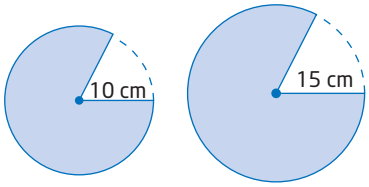


Communicate Your Understanding

C1 A cone is formed from a circle with a 90° sector removed. Another cone is formed from a semicircle with the same radius. How do the two cones differ? How are they the same?



C2 A cone is formed from a circle of radius 10 cm with a 60° sector removed. Another cone is formed from a circle of radius 15 cm with a 60° sector removed. How do the two cones differ? How are they the same?



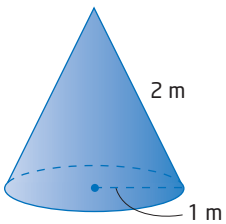
C3 The slant height of a cone is doubled. Does this double the surface area of the cone? Explain your reasoning.

Practise

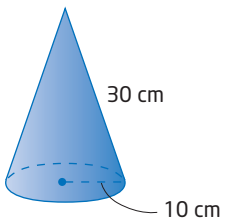
For help with questions 1 and 2, see the Example.

1. Calculate the surface area of each cone. Round to the nearest square unit.

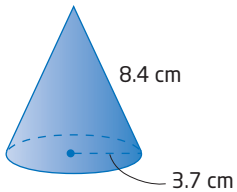
a)



b)

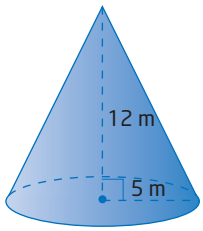


c)



2. a) Find the slant height of the cone.

b) Calculate the surface area of the cone. Round to the nearest square metre.

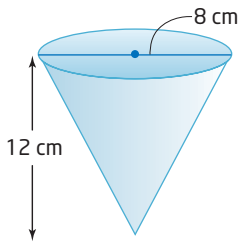


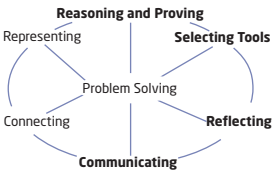
Connect and Apply

3. Some paper cups are shaped like cones.

a) How much paper, to the nearest square centimetre, is needed to make the cup?

b) What assumptions have you made?





4. One cone has base radius 4 cm and height 6 cm. Another cone has a base radius 6 cm and height 4 cm.

a) Do the cones have the same slant height?

b) Do the cones have the same surface area? If not, predict which cone has the greater surface area. Explain your reasoning.

c) Determine the surface area of each cone to check your prediction. Were you correct?

5. The lateral area of a cone with radius 4 cm is 60 cm^2.

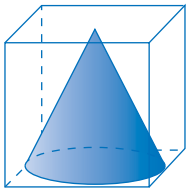
a) Determine the slant height of the cone, to the nearest centimetre.

b) Determine the height of the cone, to the nearest centimetre.

6. The height of a cone is doubled. Does this double the surface area? Justify your answer.

7. The radius of a cone is doubled. Does this double the surface area? Justify your answer.

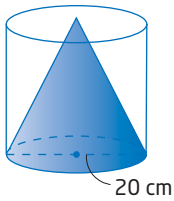
8. A cube-shaped box has sides 10 cm in length.



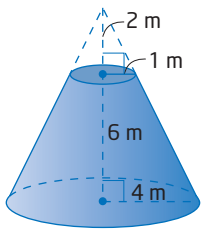
a) What are the dimensions of the largest cone that fits inside this box?

b) What is the surface area of this cone, to the nearest square centimetre?

9. A cone just fits inside a cylinder. The volume of the cylinder is 9425 cm^3. What is the surface area of this cone, to the nearest square centimetre?

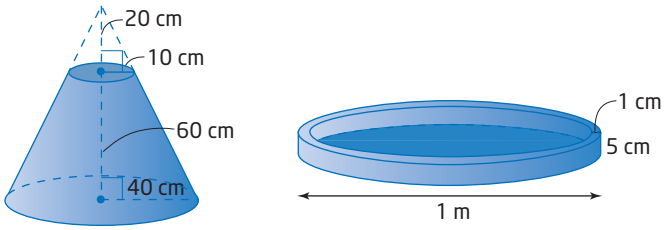


10. The frustum of a cone is the part that remains after the top portion has been removed by making a cut parallel to the base. Calculate the surface area of this frustum, to the nearest square metre.



11. **Chapter Problem** Emily has obtained an unfinished ceramic birdbath for one of her customers. She plans to paint it with a special glaze so that it will be weatherproof. The birdbath is constructed of two parts:

- a shallow open-topped cylinder with an outside diameter of 1 m and a depth of 5 cm, with 1-cm-thick walls and base
- a conical frustum on which the cylinder sits



a) Identify the surfaces that are to be painted and describe how to calculate the area.

b) Calculate the surface area to be painted, to the nearest square centimetre.

c) One can of glaze covers 1 m^2. How many cans of glaze will Emily need to cover all surfaces of the birdbath and the frustum?

12. Create a problem involving the surface area of a cone. Solve the problem. Exchange with a classmate.

Extend

13. Suppose the cube in question 8 has sides of length x.

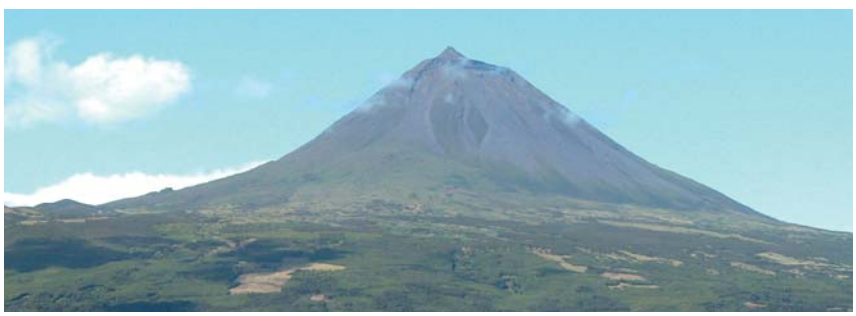
a) Write expressions for the dimensions of the largest cone that fits inside this box.

b) What is a formula for the surface area of this cone?

14. **a)** Find an expression for the slant height of a cone in terms of its lateral area and its radius.

b) If the lateral area of a cone is 100 cm^2 and its radius is 4 cm, determine its slant height.

15. Located in the Azores Islands off the coast of Portugal, Mt. Pico Volcano stands 2351 m tall. Measure the photo to estimate the radius of the base of the volcano, and then calculate its lateral surface area, to the nearest square metre.



Did You Know?

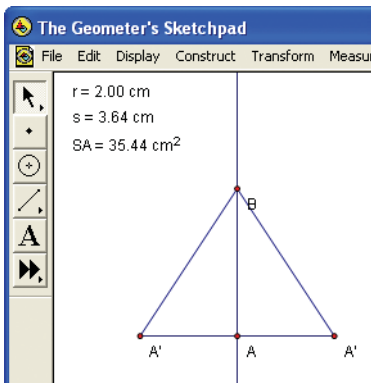
There are 8000 to 10 000 people of Azorean heritage living in Ontario.

16. Use Technology A cone has a radius of 2 cm.

a) Write an algebraic model for the surface area of this cone in terms of its slant height.

b) Use *The Geometer's Sketchpad*® to investigate how the surface area of a cone changes as the slant height changes. Since *The Geometer's Sketchpad*® cannot easily show three-dimensional objects, represent the cone with a triangle that is a side view of the cone.

- From the **Edit** menu, choose **Preferences**. Click on the **Text** tab. Ensure that **For All New Points** is checked.
- Draw a point A. Select point A. From the **Transform** menu, choose **Translate**. Ensure that the **Polar**, **Fixed Distance**, and **Fixed Angle** radio buttons are on. Change the distance to 2 cm and the angle to 0°. Click on **Translate**. Point A′ will appear 2 cm to the right of point A. Draw another point 2 cm to the left of point A, using an angle of 180°.
- Construct a line segment joining the three points. Select point A and the line segment. From the **Construct** menu, choose **Perpendicular Line** to draw a perpendicular line through point A.
- Draw a point B on the line above point A. Construct line segments to form a triangle. This triangle represents the side view of a cone with a variable height AB and a fixed radius of 2 cm.
- Measure the radius of the cone. Select this measurement. Right click and choose **Label Measurement** from the drop-down menu. Change the label to **r**.
- Measure the slant height of the cone. Change the label to **s**.
- Select **r** and **s**. From the **Measure** menu, choose **Calculate**. Enter the formula π***r**^2+π***r*****s** by selecting **π, r,** and **s** from the **Values** drop-down menu on the calculator. Change the label to **SA**. This is the surface area of the cone. Drag point B back and forth along the line. Watch how the measurements change.



- Select **s** and then **SA**. From the **Graph** menu, choose **Tabulate**. Move the table to a convenient location. Move point B, and note how the values in the table change.
- Adjust the value of **s** to about 3 cm. Select the table. From the **Graph** menu, choose **Add Table Data**. Click on **OK**. Repeat this process with **s** set to about 4 cm. Continue until you have five sets of data.

From the **Graph** menu, choose **Plot Table Data**. You will see a graph of the data that you have collected.

c) Describe the relationship that resulted from this investigation using mathematical terms.